convex skeleton: generalization of network **spanning tree**

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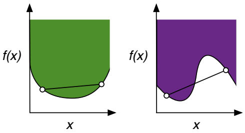
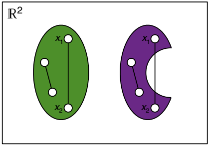
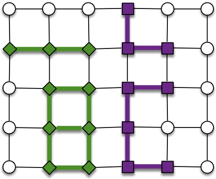
LaRichNet '18

convex subgraphs of networks

convex/**non-convex** real functions, sets in $\mathbb{R}^2$ & subgraphs







disconnected $\supseteq$ connected $\supseteq$ **induced** $\supseteq$ isometric $\supseteq$ **convex** subgraphs

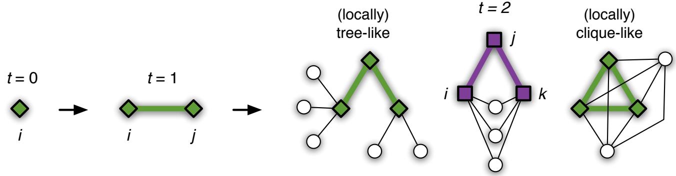
convex hull $\mathcal{H}(S)$ is smallest convex subgraph including S

subset S is convex if it induces convex **subgraph**

expansion of convex subgraphs

grow subgraph S by one node & **expand** S to convex hull $\mathcal{H}(S)$

- $S = \{\text{random node } i\}$
- until S contains n nodes:
    1. select $i \notin S$ by random edge
    2. expand $S = \mathcal{H}(S \cup \{i\})$

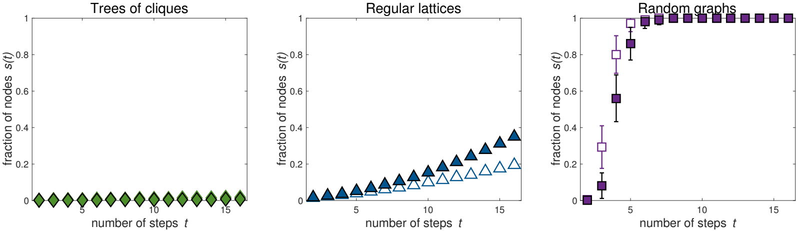


S quantifies (locally) **tree-like**/**clique-like** structure of networks

examples of convex expansion

$s(t)$ = average fraction of nodes in S after t expansion steps

$s(t) \approx (t+1)/n$ in **convex** & $s(t) \gg (t+1)/n$ in **non-convex** networks



$s(t)$ quantifies (locally) **tree-like**/**clique-like** structure of networks

measure of network convexity

$$Xs = s - \sum_{t=1}^{sn-1} \max(s\Delta s(t) - 1/n, 0) \qquad s = \text{fraction of nodes in LCC}$$

Xs highlights **tree-like**/**clique-like** networks & synthetic graphs

	n	$\langle k \rangle$	Xs
Jazz musicians	198	27.70	0.12
Network scientists	379	4.82	0.85
Computer scientists	239	4.75	0.64
Plasmodium falciparum	1158	4.15	0.43
Saccharomyces cerevisiae	1458	2.67	0.68
Caenorhabditis elegans	3747	4.14	0.56
AS (January 1, 1998)	3213	3.50	0.66
AS (January 1, 1999)	531	4.58	0.49
AS (January 1, 2000)	3570	3.94	0.59
Little Rock Lake	183	26.60	0.02
Florida Bay (wet)	128	32.42	0.03
Florida Bay (dry)	128	32.91	0.03

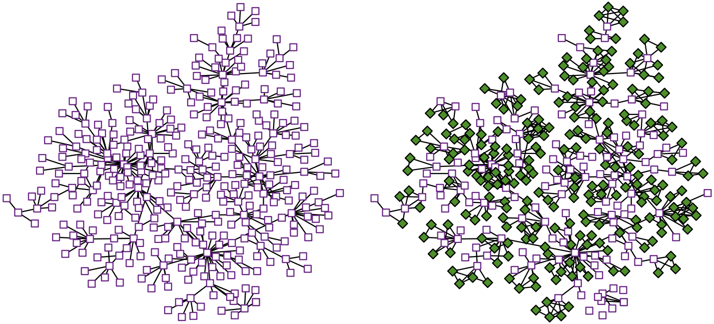
	n	$\langle k \rangle$	Xs
	2500	10.00	0.00
Random graphs	1000	10.00	0.01
	225	10.00	0.03
Triangular lattice	225	5.48	0.23
Rectangular lattice	225	3.73	0.13
Core-periphery graph	3747	4.48	0.39
	2500	5.97	1.00
Trees of cliques	1000	5.97	1.00
	225	6.01	1.00

Xs measures **global** & **regional** convexity in (disconnected) networks

convex skeletons of networks

convex skeleton = largest high-Xs subnetwork (every S convex)

spanning tree & **convex skeleton** of network scientists coauthorships



convex skeleton is **tree** of **cliques** extracted by edge removal

statistics of convex skeletons

$$\langle C \rangle = \frac{1}{n} \sum_i \frac{2t_i}{k_i(k_i - 1)} \qquad \langle \sigma \rangle = \frac{2}{n(n-1)} \sum_{i<j} \sigma_{ij} \qquad Xs = \dots$$

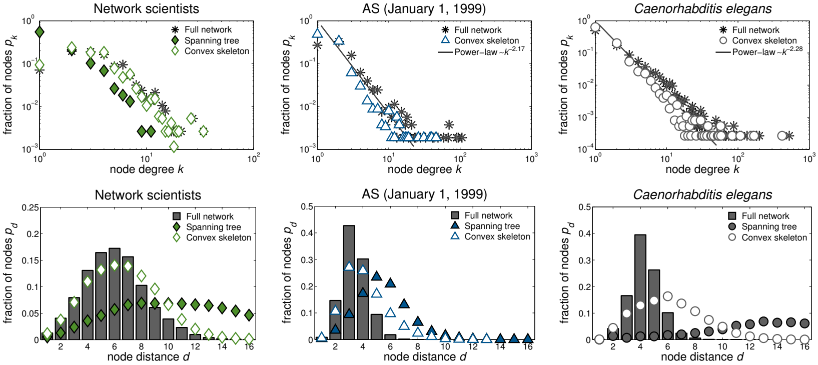
statistics of **convex skeletons** & **spanning trees** of networks

	clustering $\langle C \rangle$			geodesics $\langle \sigma \rangle$			convexity Xs		
	N	CS	ST	N	CS	ST	N	CS	ST
Jazz musicians	0.62	0.81	0.00	9.71	1.97	1.00	0.12	0.84	1.00
Network scientists	0.74	0.75	0.00	2.66	1.47	1.00	0.85	0.95	1.00
Computer scientists	0.48	0.54	0.00	4.08	1.42	1.00	0.64	0.95	1.00
Plasmodium falciparum	0.02	0.07	0.00	3.71	1.77	1.00	0.43	0.95	1.00
Saccharomyces cerevisiae	0.07	0.10	0.00	2.58	1.19	1.00	0.68	0.88	1.00
Caenorhabditis elegans	0.06	0.12	0.00	6.79	3.03	1.00	0.56	0.85	1.00
AS (January 1, 1998)	0.18	0.21	0.00	3.87	2.32	1.00	0.66	0.91	1.00
AS (January 1, 1999)	0.18	0.27	0.00	3.54	2.05	1.00	0.49	0.95	1.00
AS (January 1, 2000)	0.20	0.25	0.00	4.81	3.07	1.00	0.59	0.90	1.00
Little Rock Lake	0.32	0.69	0.00	22.13	4.32	1.00	0.02	0.82	1.00
Florida Bay (wet)	0.33	0.79	0.00	9.17	1.37	1.00	0.03	0.92	1.00
Florida Bay (dry)	0.33	0.82	0.00	9.37	1.65	1.00	0.03	0.93	1.00

convex skeleton is generalization of **spanning tree** retaining **clustering**

distributions of convex skeletons

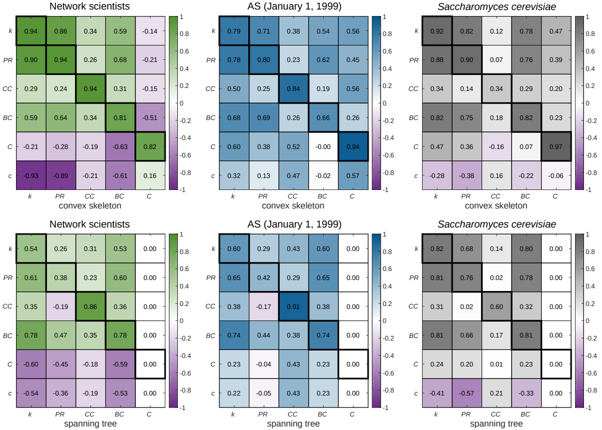
node distributions of **convex skeletons** & **spanning trees** of networks



convex skeletons retain node distributions in contrast to **spanning trees**

position in convex skeletons

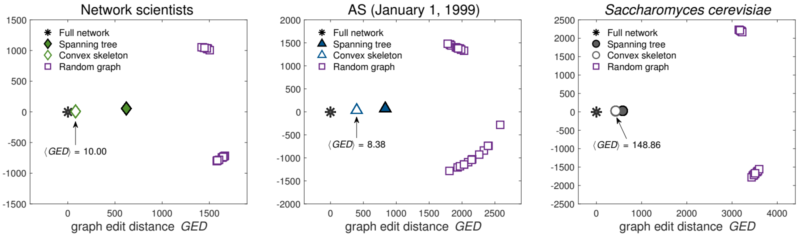
node position in **convex skeletons** & **spanning trees** of networks



convex skeletons retain node position in contrast to **spanning trees**

robustness of convex skeletons

MDS maps of **convex skeletons**, **spanning trees** & random graphs

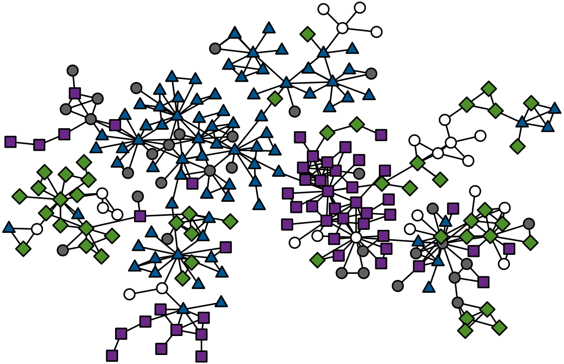


networks allow robust extraction of **convex skeletons** & **spanning trees**

convex skeleton of coauthorships

convex skeleton $\sim$ network abstraction technique

convex skeleton of Slovenian **computer scientists** coauthorships



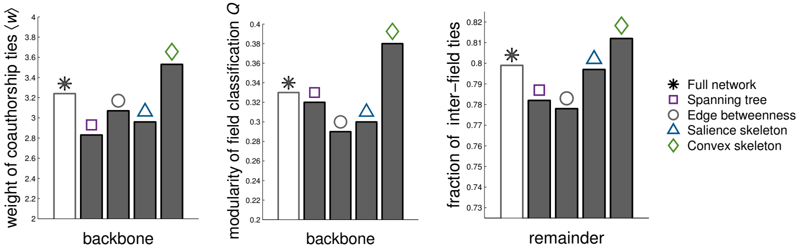
computer theory (◆), information systems (■), intelligent systems (▲),

programming technologies (○) & other (●)

network backbones of coauthorships

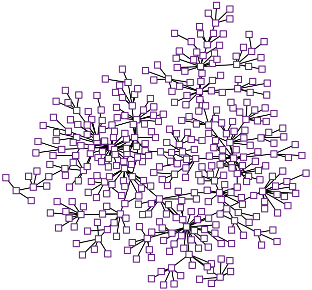
convex skeleton $\gg$ high-betweenness & high-salience skeletons

properties of **backbones** of Slovenian **computer scientists** coauthorships



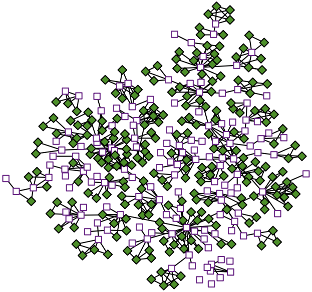
convex skeletons strengthen properties in contrast to **other backbones**

convex skeletons of networks



spanning **tree**

tree w/o cliques



convex skeleton

tree w/ cliques

convex skeleton $\gg$ network backbones

analysis, modeling, sampling, abstraction, visualization etc.

network convexity:

arXiv:**1608.03402v3**

convex skeletons:

arXiv:**1709.00255v3**

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