Three forms of convexity in graphs and networks

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Convexity is a property of a part of mathematical object that includes all the shortest paths between its units. In the case of graphs or networks, a connected subgraph is convex if every shortest path between the nodes of the subgraph lies entirely within the subgraph. We define a convex network as such in which every connected subset of nodes induces a convex subgraph. We show that such convexity is an inherent property of many networks that is not present in random graphs [1]. In particular, spatial infrastructure networks and social collaboration graphs are globally convex due to their tree/clique-like structure. Core-periphery networks are regionally convex as they can be divided into a non-convex core surrounded by a convex periphery. Random graphs, however, are merely locally convex meaning that only subgraphs of size smaller than the average distance between the nodes are convex.

1 T. Marc, L. Šubelj, *Convexity in complex networks*, arXiv:1608.03402v3, pp. 27 (2016).

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