network *representations*

introduction to *network analysis* (*ina*)

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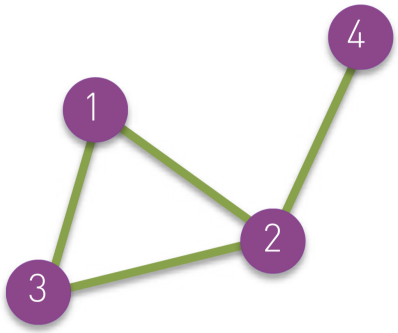
spring 2017/18

network *representations*



undirected graph

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

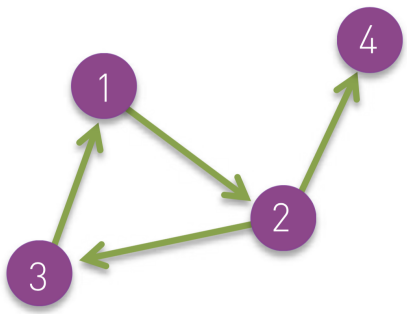
adjacency matrix

1: [2, 3]
2: [1, 3, 4]
3: [1, 2]
4: [2]

adjacency list

$\{1, 2\}$
$\{1, 3\}$
$\{2, 3\}$
$\{2, 4\}$

edge list



directed graph

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

adjacency matrix

[3] :1: [2]
[1] :2: [3, 4]
[2] :3: [1]
[2] :4: [ ]

adjacency list

$(1, 2)$
$(2, 3)$
$(2, 4)$
$(3, 1)$

edge list

---

* adjacency list can also be implemented with maps or trees & edge list cannot represent isolated nodes

network *representations*

— *adjacency matrix* for elegant *analytical derivations*

most derivations based on matrix representation†

— *adjacency list* for efficient *algorithms implementation*

ideal complexity while most algorithms require incidence†

— *edge list* for efficient *network storing/manipulation*

easy editing while each edge stored only once

† many derivations can also be based on adjacency list & some algorithms require edge list

network *structures*

— *edge list edges data structures* complexity

data structure	link manipulation	random node	random link
array	none	$\mathcal{O}(m)$	$\mathcal{O}(1)$
array list	addition	$\mathcal{O}(m)$	$\mathcal{O}(1)$
hash map	any	$\mathcal{O}(m)$	$\mathcal{O}(m)$

— *adjacency list nodes data structures* complexity

data structure	node manipulation	random node	random link
array	none	$\mathcal{O}(1)$	$\mathcal{O}(m)$
array list	addition	$\mathcal{O}(1)$	$\mathcal{O}(m)$
hash map	any	$\mathcal{O}(n)$	$\mathcal{O}(m)$

— *adjacency list neighbors data structures* complexity

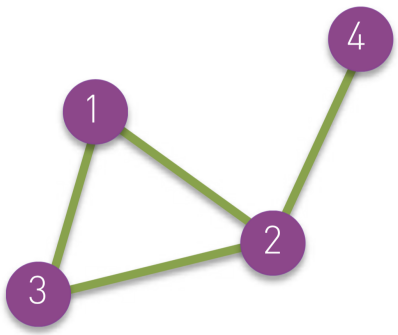
data structure	link manipulation	node incidence	random neighbor
array	none	$\mathcal{O}(k)$	$\mathcal{O}(1)$
array list	addition	$\mathcal{O}(k)$	$\mathcal{O}(1)$
hash map	any	$\approx \mathcal{O}(1)$	$\mathcal{O}(k)$
tree map	any	$\mathcal{O}(\log k)$	$\mathcal{O}(k)$

— *hash maps* for *construction* and *arrays* for *analysis*

— usually *directed adjacency list* with *undirected flag*

‡ random link selection equivalent to random node selection by degree

network *formats*



undirected graph

```
# undirect
1 2
1 3
2 3
2 4
```

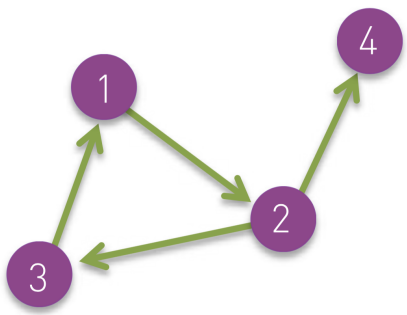
edge list

```
*vertices 4
1 "1"
2 "2"
3 "3"
4 "4"
*edges
1 2
1 3
2 3
2 4
```

Pajek format

```
# undirect
# 0 "1"
# 1 "2"
# 2 "3"
# 3 "4"
#
0 1
0 2
1 2
1 3
```

LNA format



directed graph

```
# directed
1 2
2 3
2 4
3 1
```

edge list

```
*vertices 4
1 "1"
2 "2"
3 "3"
4 "4"
*arcs
1 2
2 3
2 4
3 1
```

```
# directed
# 0 "1"
# 1 "2"
# 2 "3"
# 3 "4"
#
0 1
1 2
1 3
2 0
```

§ ad-hoc edge list and Pajek format most popular & other formats GML, GraphML and JSON proposal

network *data*

- present in many *standard datasets*
- easily obtained from *online sources*
- popular *network repositories/collections*

  KONECT ICON SNAP NetRepo Pajek LINK

- *personal web pages* of network researchers

  Mark Newman Albert Barabási Hernán Makse Emilio Ferrara Lovro Šubelj